**Waves and Oscillations**

**Periodic & Oscillatory Motion: -**

The motion in which repeats after a regular interval of time is called periodic motion.

1. The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position, is called oscillatory motion.

2. In all type of oscillatory motion one thing is common i.e each body (performing oscillatory motion) is subjected to a restoring force that increases with increase in displacement from mean position.

3. Types of oscillatory motion: - It is of two types such as linear oscillation and circular oscillation.

**Example of linear oscillation: -**

1. Oscillation of mass spring system.

2. Oscillation of fluid column in a U-tube.

3. Oscillation of floating cylinder.

4. Oscillation of body dropped in a tunnel along earth diameter.

5. Oscillation of strings of musical instruments.

**Example of circular oscillation: -**

1. Oscillation of simple pendulum.

2. Oscillation of solid sphere in a cylinder (If solid sphere rolls without slipping).

3. Oscillation of a circular ring suspended on a nail.

4. Oscillation of balance wheel of a clock.

5. Rotation of the earth around the sun.

**Oscillatory system:-**

1. The system in which the object exhibit to & fro motion about the equilibrium position with a restoring force is called oscillatory system.

2. Oscillatory system is of two types such as mechanical and non- mechanical system.

3. Mechanical oscillatory system:-

- In this type of system body itself changes its position.
- For mechanical oscillation two things are specially responsible i.e Inertia & Restoring force.
- E.g oscillation of mass spring system, oscillation of fluid-column in a U-tube, oscillation of simple pendulum, rotation of earth around the sun, oscillation of body dropped in a tunnel along earth diameter, oscillation of floating cylinder, oscillation of a circular ring suspended on a nail, oscillation of atoms and ions of solids, vibration of swings…etc.

4.Non-mechanical oscillatory system: - In this type of system, body itself doesn"t changes its position but its physical property varies periodically. e.g:-The electric current in an oscillatory circuit, the lamp of a body which is heated and cooled periodically, the pressure in a gas through a medium in which sound propagates, the electric and magnetic waves propagates undergoes oscillatory change.

# Simple Harmonic Motion

It is the simplest type of oscillatory motion.

A particle is said to be execute simple harmonic oscillation is the restoring force is directed towards the equilibrium position and its magnitude is directly proportional to the magnitude and displacement from the equilibrium position.

If F is the restoring force on the oscillator when its displacement from the equilibrium position is x, then

F –x

Here, the negative sign implies that the direction of restoring force is opposite to that of displacement of body i.e towards equilibrium position.

F= -kx ………… (1)

Where, k= proportionality constant called force.

$$Ma=-kx$$

$$M\frac{d2y}{dt2}=-kx$$

$$M\frac{d2y}{dt2}+kx=0$$

$$\frac{d2y}{dt2}+\frac{k}{M}x=0$$

$$\frac{d2y}{dt2}+\omega^2x=0 \quad \ldots\ldots\ldots\ldots (2)$$

Where $\omega^2 = \frac{k}{M}$

Here $\omega = \sqrt{\frac{k}{M}}$ is the angular frequency of the oscillation.

Equation (2) is called general differential equation of SHM.

By solving these differential equation

$x = \alpha e^{-i\omega t} + \beta e^{i\omega t}$ ……… (3)

Where $\alpha$, $\beta$ are two constants which can be determined from the initial condition of a physical system.

Appling de-Moiver's theorem

$x = \alpha(\cos\omega t + i\sin\omega t) + \beta(\cos\omega t - i\sin\omega t)$

$x = (\alpha + \beta)\cos\omega t + (\alpha - \beta)\sin\omega t$

$x = C\cos\omega t + D\sin\omega t$ ………(4)

Where $C = \alpha + \beta$

& $D = \alpha - \beta$

# Simple Hormonic motion

- Periodic motion :- repeats itself after regular interval of time. eg:- U.C.M., orbital motion etc
- Ossilatory motion (Harmonic motion) :- To & fro about fixed point after regular interval of time. eg:- pendulum, spring system
  - All ossilatory motion [with no energy lost] are periodic motion.
  - Condition for any Harmonic motion.

$$F = -kx^n$$

- $F$ → Restoring force → Always towards mean position
- $k$ → constant
- $x$ → displacement from mean position
- $n$ → (any odd integer i.e. 1, 3, 5...)

- when 'n' = 1 → Simple Hormonic motion → straight line

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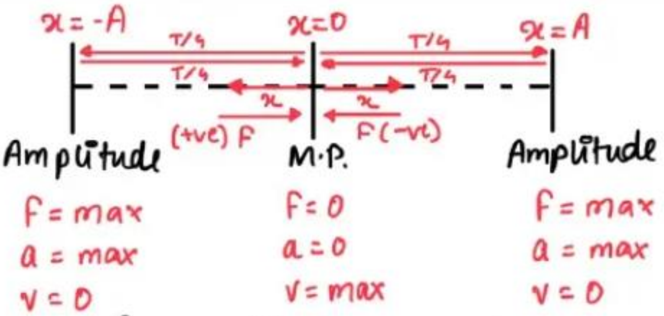
★ S.H.M.

$$\boxed{F = -kx}$$




- Amplitude (A) → maximum displacement from mean position
- Time period (T) → Time to complete 1 ossilation
- frequency (f) → Number of ossilations per second
- Angular frequency (ω) → radian / sec.

$$\boxed{T = \frac{1}{f}, \quad \omega = \frac{2\pi}{T} = 2\pi f}$$

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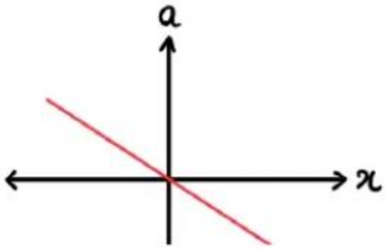
$\boxed{f = -kx}$ → for simple Hormonic motion.

- $\boxed{k = m\omega^2}$ $\therefore f = -m\omega^2 x$

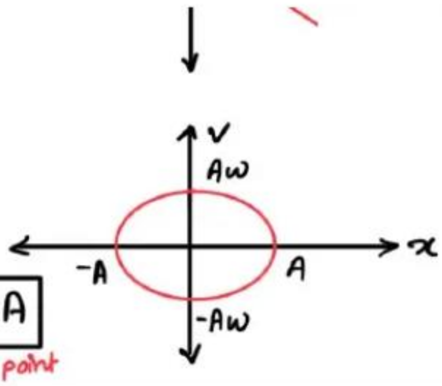
$$a = \frac{f}{m} = -\frac{m\omega^2 x}{m} \quad \boxed{\therefore a = -\omega^2 x}$$



∴ at extreme $a_{max} = -\omega^2 A$ (at extreme)

⊖ $v\frac{dv}{dx} = -\omega^2 x$ $\therefore v = \omega\sqrt{A^2 - x^2}$

→ $\frac{v^2}{(A\omega)^2} + \frac{x^2}{A^2} = 1$ (ellips), $v_{max} = \omega A$ (at mean point)

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* Differential Equation of S.H.M

(when displacement is in y-axis)

$$a = -\omega^2 x = \frac{d^2x}{dt^2} \qquad \therefore \frac{d^2x}{dt^2} + \omega^2 x = 0$$

* When a motion satisfies this equation it is Simple Harmonic motion.

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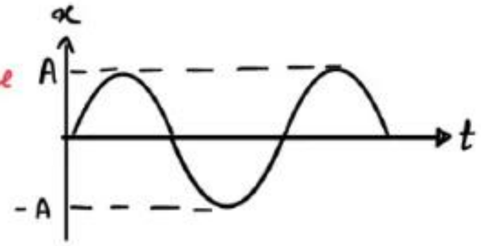
* Equation of S.H.M

⊖ $x = A\sin(\omega t + \phi)$

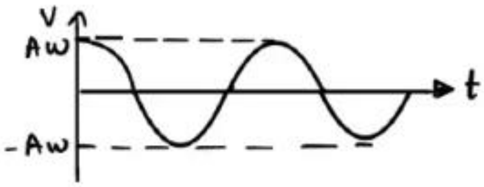
- A → Amplitude
- t → time
- $\phi$ → Initial phase
- ω → angular frequency
- x → displacement from mean position

$\phi = \sin^{-1}\left(\frac{x}{A}\right)$



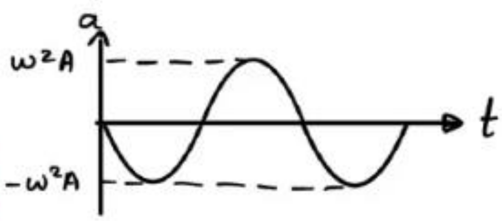

⊖ $v = A\omega\cos(\omega t + \phi)$

v → velocity




⊖ $a = -A\omega^2\sin(\omega t + \phi)$

a → acceleration




* Find $\phi$ using x & v or U.C.M when necessary
* All graphs are general i.e. when $\sin\phi = 0$ & particle starts from origin & move towards right
* Graphs will change as per the value of $\phi$ and direction of the particle

**SIMPLE PENDULUM AND PROPERTIES OF SIMPLE HARMONIC MOTION**

To investigate the dependence of time period of a simple pendulum on the length of the pendulum and the acceleration of gravity. b. To study properties of simple harmonic motion. Theory A simple pendulum is a small object that is suspended at the end of a string. "Simple" means that almost all of the system's mass can be assumed to be concentrated at a point in the object. We will use a metal bob of mass, m, hanging on an inextensible and light string of length, L, as a simple pendulum as shown in Figure 1. When the metal bob is pulled slightly away from equilibrium and released, it starts oscillating in a simple harmonic motion (SHM). The restoring force in this system is given by the component of the weight mg along the path of the bob's motion,

F = -mg sinα

and directed toward the equilibrium.

For small angle, we can write the equation of motion of the bob as

L x a = −g sinα = −g

(1) In a simple harmonic motion, acceleration is directed towards the equilibrium and proportional to the displacement. The acceleration (a) and displacement (x) are given by

$a = -\omega \; 2x$

(2) The displacement (x) varies as ( ) sin( ) x (x) t = A sin (ωt +ϕ)

(3) where ω is the angular frequency, A is the maximum displacement (amplitude) and ϕ is the initial phase of the displacement.

By comparing equations (1) and (2), for a simple pendulum, angular frequency and hence the time period of the oscillation T, are given by L g ω = ; g L T π ω π 2 2 = =

(4) The period is precisely independent of m, which reflects the fact that the acceleration of gravity is independent of m.

In this experiment, we will investigate the dependence of the period of the oscillation on L and g. By changing the length of the string, L is varied. How can we vary acceleration of gravity? In order to study properties of SHM we will use a motion detector to measure the displacement, velocity and acceleration of a pendulum with respect to times. Velocity and acceleration are given by ( ) sin( ); ( ) sin( ) 2 v a v t = Aω ωt +ϕ a t = Aω ωt +ϕ

(5) where Aω and Aω 2 are maximum velocity (Vmax) and maximum acceleration (amax) respectively, and ϕv and ϕa are phases of velocity and acceleration respectively. Figure 1. A simple pendulum x α

The maximum values of the displacement, velocity and acceleration are related as $maximum\ velocity\ (vmax)\ maximum\ displacement\ (A) = A\omega\ A = \omega\ maximum\ acceleration\ (amax)\ maximum\ displacement\ (A) = A\omega 2\ A = \omega\ 2$ (6)

and phases are related as (180 ) / 2 (90 ) o a x o v x ϕ ϕ π ϕ ϕ π − = − = (7)

The plot on the right shows an example of variation of displacement, velocity and acceleration with respect to time.

(The grapes are plotted for A = 0.5 unit, T = 10 s, and ϕx = 0) Apparatus Pendulum bob with string, support stand with clamp, stopwatch, meter stick, Vernier caliper, air table with blower, puck with string, motion sensor, Vernier Lab Quest interface device and a computer with Logger Pro software.

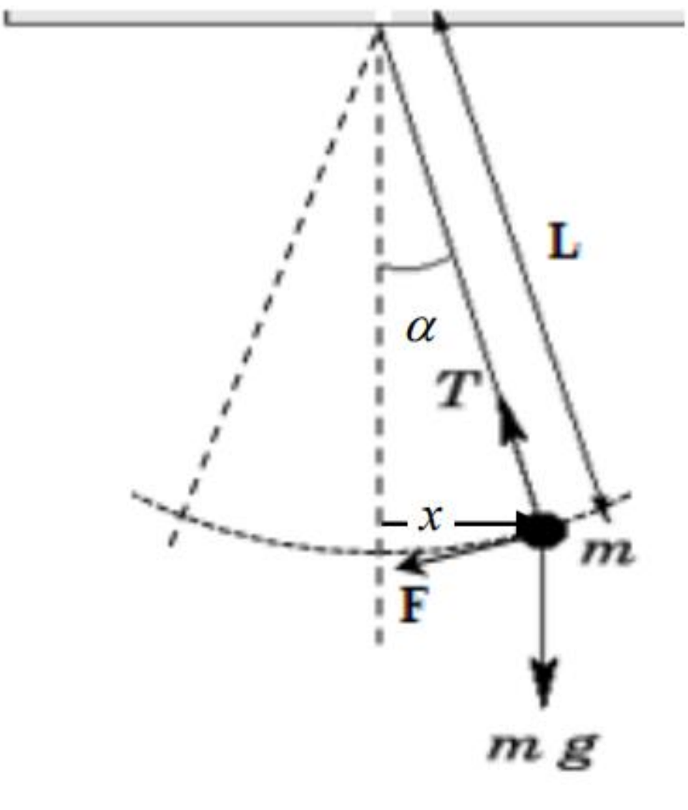



Figure 1. A simple pendulum